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Or, putting $\frac{1}{2}(1 + \cos 2\gamma)$ and $\frac{1}{2}(1 - \cos 2\gamma)$ for $\cos^2 \gamma$ and $\sin^2 \gamma$,

$$\sin (2nt - \alpha - \beta) \cos (\alpha - \beta) - \cos 2\gamma \cos (2nt - \alpha - \beta) \sin (\alpha - \beta) = 0;$$

and then replacing t by the particular value T at this time,

$$\tan (2nT - \alpha - \beta) = \cos 2\gamma \tan (\alpha - \beta).$$

3. The major semi-axis is $k \cos \delta/2$, δ being by hypothesis an angle in the first quadrant. Differentiating

$$\frac{y}{x} = \tan \gamma \frac{\cos (nt - \beta)}{\cos (nt - \alpha)},$$

we have for the angular velocity of P

$$\frac{n \tan \gamma \sin (\beta - \alpha)}{\cos^2 (nt - \alpha)},$$

positive or negative with $\sin (\beta - \alpha)$.

We can place 2γ on the hypotenuse, 2θ and δ on the legs of a right spherical triangle, $90° - (2nT - \alpha - \beta)$ opposite 2θ and $\alpha - \beta$ opposite δ on the angles; for the three equations

$$\sin \delta = \sin 2\gamma \sin (\alpha - \beta), \qquad \cos (\alpha - \beta) = \tan 2\theta \cot 2\gamma,$$
$$\cos 2\gamma = \cot (\alpha - \beta) \tan (2nT - \alpha - \beta)$$

will then conform to Napier's rules.[1]

2878 [1921, 89]. Proposed by R. S. HOAR, Fort Banks, Mass.

Consider the integers 0, 1, 2, 3 $\cdots$ $n - 1$, n. Consider all possible permutations of combinations of these taken r at a time, allowing any integer to occur more than once. Select from these permutations all groups the sum of whose integers is n. Form the reciprocal of the product of the factorials of the r integers of each of these selected groups. Then the sum of all of these reciprocals will equal $r^n/n!$. Prove that this must be so.

Example. $n = 2$, $r = 3$.

$$\frac{1}{2!0!0!} + \frac{1}{0!0!2!} + \frac{1}{0!2!0!} + \frac{1}{1!0!1!} + \frac{1}{1!1!0!} + \frac{1}{0!1!1!} = \frac{3^2}{2!}, \quad \text{if} \quad 0! = 1.$$

Solution by H. L. Olson, University of Michigan, and Philip Franklin, Princeton University.

By the multinomial theorem, we have

$$(x_1 + x_2 + \cdots + x_r)^n = \sum \frac{n!}{i_1!\, i_2! \cdots i_r!} x_1^{i_1} x_2^{i_2} \cdots x_r^{i_r},$$

where $i_1 + i_2 + \cdots + i_r = n$ and $i_1, i_2, \cdots, i_r \geqq 0$. If we set each x_i equal to unity and divide the two members of the equation by $n!$, we have the theorem stated.

2880 [1921, 89]. Proposed by SIDNEY DORB, Detroit, Mich.

Solve the simultaneous equations $xy = 2$, and

$$\left(3 - \frac{6y}{x - y}\right)^2 + \left(3 - \frac{6y}{x + y}\right)^2 = 82.$$

Solution by A. A. Bennett, University of Texas.

It is assumed for various reasons that the second equation was intended in the form

$$\left(3 + \frac{6y}{x - y}\right)^2 + \left(3 - \frac{6y}{x + y}\right)^2 = 82,$$

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[1] This supposes $\sin (\alpha - \beta)$ positive, and it would not essentially restrict the problem to assume that α and β are angles in the first quadrant, $\alpha > \beta$.

which makes the problem of interest. With this supposition, the pair of equations may be written as

$$xy = a,$$
$$\left(\frac{x+y}{x-y}\right)^2 + \left(\frac{x-y}{x+y}\right)^2 = 2b,$$

where $a = 2$, $b = 41/9$. It is then to be noted that if (r, s) be one solution, the following are the eight distinct solutions:

$$(r, s),\quad (s, r),\quad (-r, -s),\quad (-s, -r),\quad (ir, -is),\quad (is, -ir),\quad (-ir, is),\quad (-is, ir).$$

Thus there is a group of eight operations carrying any solution into a solution. This remark applies for all finite non-vanishing values of a, b. Solutions can coincide in pairs only for a, or b, zero or infinite, or for $b = -1$. Since $(1, 2)$ is by inspection a solution, the eight solutions may be written out at once.

By direct methods also the solution is readily obtained. Put $u = (x + y)/(x - y)$; whence

$$x^2 = a(u + 1)/(u - 1),$$
$$y^2 = a(u - 1)/(u + 1),$$

where the signs of x and y are to be so taken that $xy = a$. Then

$$u^2 + 1/u^2 = 2b, \qquad u = \frac{\pm\sqrt{b+1} \pm \sqrt{b-1}}{\sqrt{2}}$$

In the given example $u = 3,\ -3,\ \frac{1}{3}$, or $-\frac{1}{3}$, $x^2 = 4,\ 1,\ -4,\ -1$, $y^2 = 1,\ 4,\ -1,\ -4$, and the solutions are

$$(2, 1),\quad (-2, -1),\quad (1, 2),\quad (-1, -2),\quad (2i, -i),\quad (-2i, i),\quad (i, -2i),\quad (-i, 2i).$$

Note.—Miss Berta M. King remarks that if the sign in the second equation is changed as stated in Professor Bennett's solution, the problem is in Hawkes's *Advanced Algebra*. She gets the same results as Professor Bennett.

H. N. Carleton, A. R. Nauer, J. W. Shuman and F. L. Wilmer solved the problem as it is printed above.

2898 [1921, 228]. Proposed by J. W. CLAWSON, Ursinus College.

Four straight lines determine four triangles. It is well known that the circumcenters of these triangles lie on a circle and that the circumcircles intersect this circle in a point, called the Wallace point. It is also well known that the orthocenters of the four triangles lie in a straight line, which is perpendicular to the line on which lie the middle points of the three diagonals of the quadrilateral determined by the four given straight lines.

Prove that the centroids of the four triangles lie on a parabola whose axis is parallel to the mid-diagonal line; and that the distance from the Wallace point to the mid-diagonal line is two thirds of the distance from the Wallace point to the axis of the parabola.

Solution by the Proposer.

In the statement of the problem "two thirds" should be "three halves."

Given the four lines, let the origin be at the Wallace point. The Wallace or pedal lines of this point with respect to the four triangles are coincident. Let the y-axis be parallel to this line, its equation being, say, $x = p$. Then the perpendicular from the origin on the four given lines will meet them in four points on this line, which we may call (p, a), (p, b), (p, c), (p, d), and the equations of the four given lines will be

$$px + ay = p^2 + a^2, \text{ etc.}$$

The intersections of these lines are the six points

$$p - \frac{ab}{p},\ a + b, \text{ etc.},$$

and the mid-points of the three diagonals are

$$p - \frac{ab + cd}{2p},\ \tfrac{1}{2}(a + b + c + d), \text{ etc.},$$